To trisect an angle, for example, the angle AOC, by means of this curve, produce CO to E and draw EA. Also draw OH; then FO drawn parallel to EA makes the angle $FOC=\frac{1}{3}\angle AOC$. For since $EH=HO$, by construction of the curve, $\angle OEH=\angle EOH$. But $\angle OHA,=\angle OAH,=\angle OEH+\angle EOH=2\angle OEH$. Hence, $\angle OEH+\angle OAE,=3\angle OEH,=\angle AOC$, or $\angle OEH,=\angle FOC=\frac{1}{3}\angle AOC$.

After this department, in the last issue, was in type, we received solutions of problem 299 from Professors Scheffer, Zerr, and Greenwood. Professors Scheffer and Greenwood's solutions consisted in connecting a point, G, of the ellipse with the foci F, F'. M, the middle point of FG, is taken for the center of the circle described on the focal radius, FG, as a diameter. The line AM joining M and A, the center of the ellipse, is $\frac{1}{2}F'G$, since $AF=AF'$ and M is the middle point of FG. But $\frac{1}{2}AF'=\frac{1}{2}(2a-AF)=a-\frac{1}{2}AF$, from the definition of the ellipse. Hence, MA, the distance between the centers of the auxiliary circle and the circle described on $AF=a-\frac{1}{2}AF$, the difference of their radii. Hence the circles touch.

Dr. Zerr's solution, which was analytical, made use of the same property.

CALCULUS.

228. Proposed by **B. F. FINKEL, Ph. D., Professor of Mathematics and Physics, Drury College, Springfield, Mo.**

A sphere, radius r, is dropped into a conical vessel whose vertex angle is 60°. Find the contents of the vessel between the vertex and the sphere by means of the formula, $V=\int\int\int dx\,dy\,dz$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and the PROPOSER.

$x^2+y^2+z^2=r^2$ is the equation to the sphere, and $x^2+y^2=\frac{1}{3}(2r-z)^2$ is the equation to the cone. Eliminating z we get $y=\sqrt{(\frac{3}{4}r^2-x^2)}$.

$\therefore y=\sqrt{(\frac{3}{4}r^2-x^2)}=y'$ to $y=0$, $x=\frac{1}{2}r\sqrt{3}=x'$ to $x=0$.

$$\therefore\ v=4\int_0^{x'}\int_0^{y'}[2r-\sqrt{3}\sqrt{(x^2+y^2)}-\sqrt{(r^2-x^2-y^2)}]dxdy$$

$$=4\int_0^{x'}\Bigg[r\sqrt{(\tfrac{3}{4}r^2-x^2)}-\tfrac{1}{2}(r^2-x^2)\sin^{-1}\sqrt{\frac{\frac{3}{4}r^2-x^2}{r^2-x^2}}$$

$$-\tfrac{1}{2}\sqrt{3}\,x^2\log\left(\frac{\sqrt{(\frac{3}{4}r^2-x^2)}+\frac{1}{2}r\sqrt{3}}{x}\right)\Bigg]dx$$

$$=4(\tfrac{3}{16}\pi r^3-\tfrac{1}{64}\pi r^3+\tfrac{1}{12}\pi r^3-\tfrac{1}{6}\pi r^3-\tfrac{3}{64}\pi r^3)=\tfrac{1}{6}\pi r^3.$$

229. Proposed by **J. EDWARD SANDERS, Reinersville, Ohio.**

Solve the differential equation $d^2y/dx^2=axy$.

Solution by S. A. COREY, Hiteman, Iowa, and LEROY D. WELD, Coe College, Cedar Rapids, Iowa.

Let $y=c_0+c_1x+c_2x^2+c_3x^3+c_4x^4+c_5x^5+$ etc. ...(1).

Then $d^2y/dx^2=2c_2+6c_3x+12c_4x^2+20c_5x^3+30c_6x^4+$ etc. ...(2), and $axy=ac_0x+ac_1x^2+ac_2x^3+ac_3x^4+ac_4x^5+$ etc. ...(3).

Equating coefficients of like powers of x in (2) and (3), and reducing, we obtain by substituting in (1),

$$y=c_0\left(1+\frac{ax^3}{3!}+\frac{4a^2x^6}{6!}+\frac{4.7a^3x^9}{9!}+\frac{4.7.10a^4x^{12}}{12!}+\text{etc.}\right)$$

$$+c_1\left(x+\frac{2ax^4}{4!}+\frac{2.5a^2x^7}{7!}+\frac{2.5.8a^3x^{10}}{10!}+\frac{2.5.8.11a^4x^{13}}{13!}+\text{etc.}\right)$$

Also solved similarly by G. B. M. Zerr. Professor William Hoover did not give a solution but referred to the discussion of the general problem in Forsythe's *Differential Equations*, Chapter VII, page 217, Second Edition. A discussion of the solution of this class of differential equations, by definite integrals, is also given in Price's *Infinitesimal Calculus*, Vol. II., page 484.

MECHANICS.

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187. **Proposed by M. E. GRABER, M. A., Heidelberg University, Tiffin, Ohio.**

Find the path described by a particle acted upon by a central force, the force being directly proportional to the distance of the particle.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The equations of motion are $d^2x/dt^2+\mu x=0$ and $d^2y/dt^2+\mu y=0$.

$\therefore x=A\cos t\sqrt{\mu}+B\sin t\sqrt{\mu}$, and $y=C\cos t\sqrt{\mu}+D\sin t\sqrt{\mu}$.

$\therefore (Ay-Cx)^2+(By-Dx)^2=(AD-BC)^2$, an ellipse with center of force at center.

192 **Proposed by WILLIAM HOOVER, Ph D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

A solid sphere rolls down a trough formed by two planes which make with each other an angle 2α. Find, by the principle of *vis viva*, the expression for the time of rolling down the trough when the inclination of the trough to the horizon is β.

Solution by LEROY D. WELD, Coe College, Cedar Rapids, Iowa, and the PROPOSER.

Let C be the center of the sphere, O the center of the line joining the points of contact A and B of the trough and sphere, k the radius of gyration of the sphere about its center, r=its radius, θ=the angle through which a fixed radius in the plane of O and the edge of the angle 2ϕ, has rotated in any time t from the beginning of motion; let a plane and line, the first embracing AB, and the other passing through C, be drawn parallel to the edge of 2ϕ, both cutting a fixed horizontal plane, the line in the point D; x, y, the coördinates of C at the time t, D, the origin, and a and b the initial values of x, y; then by *vis viva*, m being the mass of the sphere,

$$m\left(\frac{dx^2}{dt^2}+\frac{dy^2}{dt^2}+k^2\frac{d\theta^2}{dt^2}\right)=C-2mgy.$$